QUEUING MODEL CASE STUDY FOR A MESS

1.K.PRAKASHRANI, HOD/Associate Professor,2. S. KANISELVI, Assistant Professor
1.Department of Mathematics, A.D.M.College for women, Nagapattinam
2. Department of Mathematics ,Sri Sarada Mahavidyalayam ,Arts & Science College for Women, Ulundurpet, Villupuram District.
1. prakashranik@gmail.com,2.jagadeeshkani@gmail.com

***Abstract*: Queuing is a common activity of customers or people to avail the desired service,which could be processed or distributed one at a time. Mess or hotels would avoid losing their customers due to a long wait on the line. Some mess initially provide waiting chairs, However, waiting chairs alone would not solve a problem alone when customers withdraw the orders; the service time may need to be improved. This paper aims to show that queuing theory satisfies the model when tested with a real-case scenario. We collected the data from a mess in Ulundurpet .Using M/M/1 queuing model, we derived the arrival rate, service rate, utilization rate. Average number of customers in the mess is also obtained.**

Keywords : Arrival Rate , Service Rate , Waiting Lines , Utilization Rate .

I.INTRODUCTION

Queuing theory had its beginning in the research work of a Danish Engineer named A.K. Erlang . In 1909, Erlang experimented with fluctuating demand in telephone traffic, Eight years later, he published a report addressing the delays in automatic dialing equipment. At the end of world war II, Erlang's early work was extended to more general problems and business applications of waiting lines. Waiting line is defined as the customers who cannot be served immediately have to queue for service.

Researchers have previously used queuing theory to model the restaurant operation[1], reduce cycle time in a busy fast food restaurant[2,3], as well as to increase throughput and efficiency[4,1].

The study of waiting line, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques. Waiting lines are an everyday occurrence, affecting people shopping for groceries buying gasoline, making a bank deposit, or waiting on the telephone for the first available airline reservations to answer. Queues, another term for waiting lines, may also take the form of machines waiting to be repaired, trucks in line to be unloaded, or aeroplanes lined up on a runway waiting for permission to take off. The three basic components of a queuing process are arrivals, service rate, and the actual waiting line.

Waiting line is what one experience everywhere in daily life i.e. while shopping, checking into hotels, at hospitals and clinics etc. In situations where facilities are limited and cannot satisfy the demand made upon them, bottlenecks occur which manifest as queues, but customers are not interested in waiting in queues. When customers wait in queue, there is the threat that excessive waiting time will lead to the loss of some customers to competitors.

II. QUEUING MODEL AND KENDALL'S NOTATION

Queuing model can be characterized by the following factors:

- **Arrival time distribution**: Inter-arrival times most commonly fall into one of the following distribution patterns: a Poisson distribution, a Deterministic distribution, or a general distribution. However inter arrival times are most often assumed to be independent and memory less, which is the attributes of a Poisson distribution.
- **Service time distribution**: The service time distribution can be constant, exponential , hyper exponential, hypo- exponential or general. The service time is independent of the inter-arrival time.
- **Number of servers**: The queuing calculations change depends on whether there is a single server or multiple servers for the queue. A single server queue has one server in the system. This is the situation normally found in a grocery store where there is a line for each cashier. A multiple server queue corresponds to the situation in a Bank in which a single line waits for the first of several tellers to become available.
- **Queue lengths**: The queue in a system can be modeled as having finite or infinite queue length.
- **System capacity**: The maximum number of customers in a system can be from 1 upto infinity. This includes the customers waiting in the queue.
- **Queue discipline**: There are several possibilities in terms of the sequences of customers to be served such as FIFO (First in First out, (i.e) in order of arrival) random order, LIFO (Last in First out (i.e) the last one to come will be the first to be served), or priorities.

Kendall in 1953 proposed a notation system which describes the characteristics of queue. It can be written as

A/B/P/Q/R/Z

where A,B,P,Q,R and Z describe the queuing system properties.

- A describes the distribution type of the inter arrival times.
- B describes the distribution type of the service times.
- P describes the number of servers in the system.
- Q describes the maximum length of the queue.
- R describes the size of the system population.
- Z describes the queuing discipline.

III QUEUING SYSTEM VARIABLES

- λ= the arrival rate
- μ = the service rate
- $\lambda < \mu$ (customers are served at a faster rate than they arrive). Customers must be served faster they arrive , or an infinitely large queue will build up.
- L_q = average number of customers in queue
- L = average number of customers in system, including those being served.
- W_q= average time a customer spend in queue.
- W = average time a customer spends in queue plus service
- $L_q = \lambda W_q$ (Little's Formula[5])
- $L = \lambda W$ (Littles's Formula)
- P_0= probability of zero customers in the mess $= (1 - \lambda / \mu)$ and
- $P_n = (\lambda / \mu)^n P_0$= probability of n customers in the mess.

IV DATA AND METHODOLOGY

Method of data collection : The study was conducted at a mess in Ulundurpet through interview with the mess manager also through observation of the mess. The data was collected during the period of one month.

There are several determining factors for a mess to be considered a good or a bad one. Taste, Cleanliness, the mess layout and settings are some of the most important factors, when managed carefully, will be able to attract plenty of customers. However, there is also another factor that needs to be considered especially when the mess has already succeeded in attracting customers. This factor is the customers wating time.

The daily number of customers was obtained from the mess itself. The mess has been recording the data as part of its end of day routine. We also interviewed the mess manager to find out about the capacity of mess, the number of servers and the number of sweepers. Based on the interview with the mess manager, we concluded that this can be applied as M/M/1 model. The arrival and service time are exponentially distributed (Poisson process). The mess consists of only one server.

V RESULT AND DISCUSSION

The one month daily customer data were shared by the mess manager as shown in below:

TABLE 1 MONTHLY CUSTOMER COUNTS

Week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1st week	108	114	120	138	52	48	45
2nd week	108	118	137	126	45	46	40
3rd week	106	113	116	114	49	38	35
4th week	110	112	118	107	53	40	32

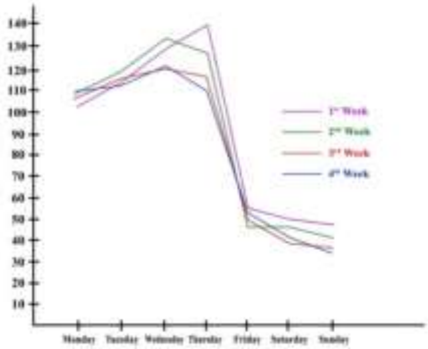

Fig. 1 : ONE MONTH DAILY CUSTOMER COUNTS

In fig.1, the number of customers on Monday to Thursday are double the number of customers during Friday to Sunday. Hence, we will focus our analysis in this time window.

VI CALCULATION

We conducted the research at dinner time. There are an average 100 people are coming to the mess in 3 hrs time window of dinner time. From this we can derive the arrival rate as :

$$\lambda = 0.83 \text{ customers/minutes}$$

We also found out from observation and discussion with manager that each customer spend 35 minutes on average in the restaurant (W), the queue length is around 10 people(Lq) on average and the waiting time is around 15 minutes.

The observed actual waiting time does not differ by much when compared to the theoretical waiting time as shown below.

$$W_q = \frac{10 \text{ customers}}{0.83 \text{ cpm}} = 12.05 \text{ minutes}$$

The average number of people in the mess is

L = 0.83 cpm x 35 minutes = 29.05 customers.

The Service rate and utilization rate are calculated as

$$\mu = \frac{\lambda(1+L)}{L} = \frac{0.83\,(1+29.05)}{29.05} = 0.8586 \text{ cpm}$$

$$\rho = \frac{\lambda}{\mu} = \frac{0.83}{0.86} = 0.965$$

With the very high utilization rate is 0.965 during dinner time. The Probability of zero customers in the mess is very small as can be derived using

$$P_o = 1 - \rho$$
$$= 0.035$$

The generic formula that can be used to calculate the probability of having n customers in the mess is an follows:

$$P_n = (1 - 0.965)\,(0.965)^n$$
$$P_n = (0.035)\,(0.965)^n$$

We assume that potential customers will start to balk when they see more than 10 people are already queuing for the mess. We also assume that the maximum queue length that a potential customer can tolerate is 10 people. As the capacity of the mess when fully occupied is 30 people. We can calculate the probability of 10 people in the queue as the probability when there are 40 people in the system.

(i.e., 30 in the mess and 10 or more queuing) as follows:

Probability of customers going away

= P(more than 10 people in the queue)

= P(more than 40 people in the mess)

$P_{11-21} = \sum_{n=11}^{21} P_n = 0.2376 = 23.76\%$

VII EVALUATION

- The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increases.
- The utilization at the mess is very high at 0.965. This , however, is only the utilization rate during lunch and dinner time on Monday to Thursday. On Friday to Sunday, the utilization rate is almost half of it. This is because the number of visitors on Friday to Sunday is only half of the number of visitors on Monday to Thursday. In addition, the number of waiters remains the same regardless whether it is peak hours or off peak hours.
- In case the customers waiting time is lower or in other words. We waited for less than 15 minutes, the number of customers that are able to be served per minute will increase. When the service rate is higher the utilization will be lower which makes the probability of the customers going away decreases.

VIII BENEFITS

- This research can help the mess to increase their QoS (qualityof service),by anticipating if there are many customers in the queue.
- The result of this paper work may become the reference to analyze the current system and improve the next system. Because the mess can now estimate of how many customers will wait in the queue and the number of customers that will go away each day.
- By anticipating the huge number of customers coming and going in a day, the mess can set a target profit that should be achieved daily.
- The formulas that were used during the completion of the research is applicable for future research and also could be use to develop more complex theories.

IX. CONCLUSION

This research paper has discussed the application of queuing theory of non-vegetarian mess. From the result we have obtained that the rate at which customers arrive in the queuing system is 0.83 cpm and the service rate is 0.86cpm. The probability of buffer flow if there are 10 or more customers in the queue.The Probability of buffer overflow is the probability that customers will run away, because may be they are impatient to wait in the queue. The theory is applicable for the mess if they want to calculate all the data daily. It can be concluded that the arrival rate will be lesser and the service rate will be greater if it is on Friday to Sunday since the average number of customers is less as compared to those on Monday to Thursday. The constraints that were faced for the completion of this research were the inaccuracy of result .since some of the data that we use was just based on assumption or approximation. We hope that this research can contribute to the betterment of mess in terms of its way of dealing with customers.

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BIOGRAPHY

First Author : K.Prakash Rani is an Associate Professor & Head in PG and Research Department of Mathematics, A.D.M. College for Women (Autonomous), Nagapattinam. She received her B.Sc. degree in Mathematics from Madras University, M.Sc.Applied Mathematics from Bharathidasan University and M.Phil. degree from (MIT), Anna University. In 1986, she joined as a Research Assistant in the Department of Mathematics, REC,Tiruchirappalli then in 1987 she joined as a Lecturer in A.D.M. College. She received her Ph.D.degree from Bharathidasan University in 2008. Her Research interests include Stochastic Processes, Queues, Retrial Queues and Graph theory. She has more than 30 years of teaching, 16 years of Research experience and she published 15 articles. Presented paper in different conferences held at Vijayawada, New Delhi and Calcutta University. She has finished one UGC sponsored Minor Research Project and she is a co-Guide for five Research Scholars.

Second Author: S.Kaniselvi is an Assistant Professor, Department of Mathematics, Sri Sarada Mahavidyalayam Arts &Science College for Women, Ulundurpet. She has Completed her B.Sc., Mathematics from A.D.M.College for women, Bharathidasan University, M.Sc., Mathematics from A.D.M.College for Women, Bharathidasan University and M.Phil., Degree from Madurai kamaraj University. She has five years teaching experience from Sri Sarada Mahavidyalayam Arts and Science College for Women, Thiruvalluvar University. She is doing Research now.Her Research interests include Stochastic Process, Queues, Retrial Queues & Graph theory.