FUZZY CONJOINT MODEL IN MEASURING STUDENTS' EXPECTATION AND TEACHERS' BELIEFS ON LEARNING MATHEMATICS

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***Abstract:** The quality of teaching and learning mathematics have been one of the major challenges of educators. Mathematics is the subject that has the lot of skills, concepts with different kinds of topics. This paper deals with to measure the students' expectations and teachers confident on learning mathematics using fuzzy conjoint model. This study employed a fuzzy approach questionnaire which consists of six attributes for measuring students' expectations and other six for teachers' confident. The fuzzy conjoint model is based on fuzzy set theory and it is used to analyze the data from 243 under graduate mathematics students and 27 professors from Thiruvarur district, Tamilnadu. The scores of students' expectations and teachers' beliefs were changed into degree of similarity and level of agreement for attributes using fuzzy conjoint model. These similarity measures are used for measuring teachers' beliefs and students' expectations on learning mathematics.*

***Keywords**: Conjoint Analysis, Degree Of Similarity, Membership Function, Learning Mathematics.*

I INTRODUCTION

The learning and teaching mathematics are both complex active process.[1].Teachers are making decisions to motivate the students as active learners. In total education of each student, the teachers must undertake long term planning to connect daily efforts. At the same time, understanding of pure mathematics papers have always been challenging criteria for the mathematics students.

Theory papers for mathematics majors are typically taught in a lecture format, which is included of presenting definitions, theorems, and proofs. There is a general cognition between teachers and students that these lectures are not as effective as they could be. However, the issues of why these lectures are not effective and how they might be improved are not discussed often in the mathematics education literature. Elices lockwod sought that [2], in task-based interview, with students are discussed about pedagogy with mathematicians, observed the reactions. From that he analyzed that teachers and students have different expectations of lectures and these different expectations lead to barriers in communication.

By expectations, referred to (i) what a student is supposed to learn from a lecture and (ii) how students should engage in the lecture to understand this content. In this paper, we have described six attributes for teachers' belief and another six for student expectations, with the hope that discussing these differing expectations might help us to improve our students learning mathematics skill at the undergraduate level.

The perceptions and learning are very subjective and analyze the learning attributes using statistics. Perceptions toward learning attributes are subjective indeed. Therefore, an alternative method based on fuzzy approach is possible. At this point, it is suitable to fuzzy set theory being applied. The uncertainty or vagueness and inherent in the definition of linguistic variables are represented by fuzzy sets by Zimmerman [3]. The development of fuzzy set theory divvied into two different categories [4]. Firstly, the fuzzy set theory is a formal theory which extends in the scope of the classical mathematical area such as algebra, graph theory, and topology. Secondly, the fuzzy set theory is a powerful modeling language that can undertake with a huge fraction of uncertainties in real life situations. Fuzzy set theory has been applied to many areas such as social science, medical, finance, management, marketing and psychology as well. Fuzzy sets are representing the linguistic variables for substantial research in cognitive psychology. Weon and Kim applied fuzzy membership function in learning achievement evaluation [5]. In our previous research papers [6-12], we describe the application of fuzzy set in teachers' performance and students' learning skill on mathematics. In this paper, we use fuzzy set theory in education to focus on teachers' confident and students' expectations on mathematics lecture. The fuzzy set conjoint model is purely based on the fuzzy sets preference model.

This paper divided into five sections. The following section proposes how to construct the research questions. The fuzzy conjoint model is discussed in the third section. The fourth section explains the proposed fuzzy modeling with a suitable case study. Finally the conclusion is given.

II RESEARCH QUESTIONS

In this research, mathematics teachers' confident and students' expectations about learning mathematics are measured by using the fuzzy conjoint model. Green and Srinivasan worked conjoint analysis in market research [13] and Turksen & Wilson have modified the conjoint approach using fuzzy set to develop a fuzzy preference model for consumer choices and market share [14]. We refer [15-18] few papers handling fuzzy conjoint

model for evaluation of bank sector and psychology skill. The measurement included degree of consensus agreement in a preference questionnaire for each selected attribute. There are many attributes constitute in a systems of belief and expectations. There are six attributes ($A_1 - A_6$) are conjoint model, the degree of similarity was used rather than the degree of consensus. Specifically, the research questions are

1. To what degree of similarity, mathematics students' expectations from mathematics lecture that

A_1: Smooth relationship - Students may convey their ideas freely.

A_2 : Conceptual understanding - Lecture should be focused to develop and understanding of concepts and procedures through problem solving.

A_3 : Motivation -Teacher provides students with appropriate challenge, encourages perseverance in solving problems and supports productive struggle in learning mathematics.

A_4: Explain mathematical modeling - How to apply mathematics in real world life.

A_5: Syllabus covered.

A_6: Interaction.

2. To what degree of similarity, mathematics teachers' belief about learning mathematics that

A_7: Mathematics is a difficult subject.

A_8: Mathematics is not numbers and symbols but a way of thinking using symbols and equations to represent real world problems in a mathematical model.

A_9: Practioning continuously with increases variations can lead to deep understanding of mathematics.

A_{10}: Drills and practice is one of the best ways of learning Mathematics.

A_{11}: Student can learn mathematics through exploring and Solving contextual and mathematical problems.

A_{12}: Student should be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge of familiar contexts.

The questionnaires are asked to 243 final years under graduate mathematics students from R.A. College for women, Thiruvarur District, Tamilnadu and S.T.E.T. women's college, Mannargudi, Thiruvarur District, Tamilnadu and 27 mathematics lecturers from the same colleges. All of the mathematics lecturers had at least three years experience in teaching mathematics at college level. The preference data are collected and analyzed using the fuzzy conjoint model.

III EVALUTION METHOD

A. Fuzzy Preference Model

Preference model are applied in market management and market segmentation [13, 19]. However, sought to be analyzed for students' expectations and other six ($A_7 - A_{12}$) for teachers' belief. In order to suit with the fuzzy

the subjective nature in element of preference has great plenty of vague, uncertainty. In both psychological attributes and the subject preferences, the linguistic variables are found everywhere. There is a vagueness of the rating 'agree' due to lack of knowledge about the available rating. The fuzzy set for 'strongly agree' consist domain variable and a degree of membership. A membership function assigns [0 ,1] for each value of the domain variable..

In fuzzy preference model, fuzzy membership function for each of linguistic variables are ratings on the measurement scale. In this research, Likert scale was applied for all preference ratings, and provide subjective with a balanced selection of 7 linguistic terms to indicate their teachers' belief and students' expectations about learning mathematics. Three positive and three negative evaluations and a central neutral evaluation are presented in this scale. The underlying theory of fuzzy sets in the preference modeling could be restoring further from [13]. Representing linguistic variables in a mathematical structure is derived from fuzzy sets in preference model that closely corresponds to actual subject preferences. The preference for a statement can be separated into a combination of preferences for its attributes, which are combined using a combination function. In the fuzzy conjoint model, the combination of preferences becomes the main underlying philosophy.

B.Fuzzy Conjoint Model

Turksen & Willson applied fuzzy set to develop an extended conjoint model which was known as The Fuzzy Conjoint Model [14]. A fuzzy set R is formed to represent the hierarchy of all respondents against the specific attributes. This approach gives a degree of consensus agreement for each selected attribute that was used in this study.

The approximate degree of membership for each element,$y_j (j = 1,2\ 3 \ldots l)$ in fuzzy set Ris defined as

$$\mu_R(y_j, M) = \sum_{i=1}^{n} W_i \, \mu_R(x_j, M) \text{ ----------(1)}$$

Where

$\mu_R(x_j, M)$: Degree of membership of domain element x_j in the subject's linguistic rating R of the i[th] attribute of M for each element in the fuzzy set

$R_i, x_j, j = 1,2..l.$

$R_i \in$ {Very strongly agree (L_1), strongly agree (L_2), agree (L_3), indifferent (L_4), disagree (L_5), strongly disagree (L_6), very strongly disagree (L_7)} by the ith respondent, i = 1 to 243 against the attribute M ($A_1 - A_6$) and i = 1 to 27 against the attribute M ($A_7 - A_{12}$).

W_i: Weight for the ith respondent$W_i = \frac{w_i}{\sum_{i=1}^{n} w_k}$, as w_i is a score of linguistic values given by the ith respondent.

l : number of linguistic values used (here, l = 7)

$\mu_R(y_j, M)$: approximate overall degree of membership of the

linguistic value R for all factor M attributes based on the domain value.
M: factor attributes
n: number of respondents.

C. Linguistic Variables

The application of linguistic variables is introduced by Zadeh [3]. In this paper, there were used 7 linguistic variables and defined as L_k = {very strongly agree (L_1), strongly agree (L_2), agree (L_3), indifferent (L_4), disagree (L_5), strongly disagree (L_6), very strongly disagree (L_7)}. Each linguistic variable are represented by fuzzy sets and are defined as follow:

Very strongly agree, L_1 = {1/1, 0.8/2, 0.5/3, 0.2/4, 0/5,0/6,0/7}
Strongly agree, L_2= {0.7/1, 1/2, 0.6/3, 0.4/4, 0/5, 0/6, 0/7}
Agree, L_3 = {0.4/1, 0.6/2, 1/3, 0.6/4, 0.4/5, 0/6, 0/7}
Indifferent, L_4 = {0/1, 0.3/2, 0.7/3, 1/4, 0.7/5, 0.3/6, 0/7}
Disagree, L_5 = {0/1, 0.2/2, 0.4/3, 0.6/4, 1/5, 0.6/6, 0.4/7}
Strongly disagree, L_6 = {0/1, 0/2, 0/3, 0.4/4, 0.6/5, 1/6, 0.7/7}
Very strongly disagree, L_7 = {0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.8/6, 1/7}

D. Degree of Similarity

For each attribute, M the calculation of degree of similarity between the fuzzy set representing the whole respondents with every fuzzy set that represented by 7 linguistic values (L_k,k = 1,2,3,4,5,6,7) was executed using a distance formula. The degree of similarity was defined as the Euclidean distance between fuzzy set R and L_k. The formula for the similarity of two sets is

$$S_i(R, L_K) = \frac{1}{[1+\sqrt{\sum_{j=1}^{7}(\mu_R(j,M)-\mu_{L_k}(j))^2}} \qquad (2)$$

E. Measurement Procedures

There are 12 attributes to be considered and analyzed in this paper. The computations are executed based on the following steps

Step 1: Collect the students' and teachers' responses, R (i = 1,2,3,….n).
Step 2: Calculate the weights for each respondent, $W_i = \frac{w_i}{\sum_{i=1}^{n} w_k}$.
Step 3: Find membership of each element in fuzzy set R using equation (1).
Step 4: Find similarity degree between fuzzy set R and fuzzy set L, k = 1,2,…7 using equation (2).
Step 5: Choose the linguistic L which registered the highest degree of similarity.

IV RESULTS AND DISCUSSION

Measurements and discussions for each attribute were presented in accordance with the respective category. The students' and teachers' opinions related to proposed attributes, fuzzy output vectors, respective similarity degree are given in the following tables.

TABLE: 1
STUDENTS' OPINIONS RELATED TO PROPOSED ATTRIBUTES

Attributes	L_1	L_2	L_3	L_4	L_5	L_6	L_7	Total
A_1	24	36	97	48	38	0	0	243
A_2	121	73	48	0	0	0	0	242
A_3	74	49	61	40	19	0	0	243
A_4	53	34	89	38	14	6	9	243
A_5	68	110	34	30	0	0	0	242
A_6	40	40	108	44	6	3	2	243

TABLE: 2
OUTPUT FUZZY VECTORS FOR STUDENTS' PROFILE WITH RESPECT TO LINGUISTIC VARIABLES

Fuzzy set	L_1	L_2	L_3	L_4	L_5	L_6	L_7
R_1	0.09876	0.14815	0.39918	0.19753	0.15638	0	0
R_2	0.5	0.30165	0.19835	0	0	0	0
R_3	0.30453	0.20165	0.25103	0.16461	0.07819	0	0
R_4	0.21811	0.13992	0.36626	0.15638	0.05761	0.02469	0.03704
R_5	0.28099	0.45454	0.14049	0.12397	0	0	0
R_6	0.16461	0.16461	0.44444	0.18107	0.02469	0.01234	0.00823

TABLE: 3
VALUES OF SIMILARITY DEGREE BETWEEN THE FUZZY SET R AND LINGUISTIC VARIABLES L(STUDENTS' EXPECTATIONS)

Fuzzy set	L_1	L_2	L_3	L_4	L_5	L_6	L_7
R_1	0.46998	0.47796	0.51637	0.48223	0.45750	0.41925	0.41734
R_2	0.55763	0.52045	0.47098	0.40937	0.40374	0.39276	0.39683
R_3	0.51164	0.5029	0.49729	0.45128	0.43593	0.41336	0.41327
R_4	0.49129	0.48507	0.50479	0.45928	0.44157	0.41756	0.41871
R_5	0.53241	0.53417	0.48138	0.42957	0.41681	0.40099	0.40264
R_6	0.48742	0.49301	0.51737	0.46533	0.43729	0.40919	0.40934

TABLE: 4
MAXIMUM SIMILARITY DEGREE (MSD) AND RANKING FOR STUDENTS' EXPECTATIONS

Fuzzy set R	R_1	R_2	R_3	R_4	R_5	R_6
MSD	0.51637	0.55763	0.51164	0.50479	0.53417	0.51737
Linguistic variable L	Agree	Very Strongly agree	Very strongly agree	Agree	Strongly agree	Agree
Ranking	4	1	5	6	2	3

TABLE: 5
TEACHERS' OPINIONS RELATED TO PROPOSED ATTRIBUTES

Attributes	L_1	L_2	L_3	L_4	L_5	L_6	L_7	**Total**
A_7	1	2	4	6	10	3	1	27
A_8	12	6	6	3	0	0	0	27
A_9	18	5	4	0	0	0	0	27
A_{10}	17	7	3	0	0	0	0	27
A_{11}	4	9	12	2	0	0	0	27
A_{12}	4	8	15	0	0	0	0	27

TABLE: 6
OUTPUT FUZZY VECTORS FOR TEACHERS' PROFILE WITH RESPECT TO LINGUISTIC VARIABLES

Fuzzy set	L_1	L_2	L_3	L_4	L_5	L_6	L_7
R_7	0.03704	0.07407	0.14815	0.22222	0.37037	0.11111	0.03704
R_8	0.44444	0.22222	0.22222	0.11111	0	0	0
R_9	0.66667	0.18519	0.14815	0	0	0	0
R_{10}	0.62963	0.25926	0.11111	0	0	0	0
R_{11}	0.14815	0.33333	0.44444	0.07407	0	0	0
R_{12}	0.14815	0.29630	0.55556	0	0	0	0

TABLE: 7
VALUES OF SIMILARITY DEGREE BETWEEN THE FUZZY SET R AND LINGUISTIC VARIABLES L (TEACHERS' BELIEFS)

Fuzzy set	L_1	L_2	L_3	L_4	L_5	L_6	L_7
R_7	0.43195	0.43525	0.46812	0.48723	0.50118	0.46234	0.45386
R_8	0.53968	0.51366	0.48268	0.42692	0.41490	0.40108	0.40302
R_9	0.55309	0.49581	0.45041	0.39519	0.39287	0.38692	0.39077
R_{10}	0.55931	0.50606	0.45124	0.39608	0.39411	0.38809	0.39198
R_{11}	0.50479	0.51606	0.5160	0.44739	0.42448	0.39805	0.40723
R_{12}	0.49722	0.50488	0.51694	0.43977	0.4182	0.39088	0.39589

TABLE: 8
MAXIMUM SIMILARITY DEGREE (MSD) AND RANKING FOR TEACHERS' BELIEFS

Fuzzy set R	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}
MSD	0.50118	0.53968	0.55309	0.55931	0.51606	0.51694
Linguistic variable L	Disagree	Very Strongly agree	Very strongly agree	Very strogly agree	Strongly agree	Agree
Ranking	6	3	2	1	5	4

From the above table 4, we found that the students' expectations are ranking with their highest degree of similarity. Firstly, the students are very strongly agreed the need of conceptual understanding of learning mathematics. Secondly, they strongly agreed for syllabus covering and then agreed interaction. Here, the highest similarity degree of each attributes are differing with very smallest value and all the attributes have the linguistic variables very strongly agree, strongly agree, agree. So the lecture will covered all those things, the classes will be very effective.

Maximum Similarity Degree (MSD) and ranking for

Teachers' belief about learning mathematics is shown in table 8. The measurement showed that the teachers very strongly agreed that 'drill and practice' is one of the best ways of learning mathematics. They are also very strongly agreed that continuous work in doing mathematics will lead to better understand of mathematics and mathematics symbols can be pursued to the mathematical modeling as the parts of the real phenomena. They agreed that mathematics

do not require much memorization compared to other sciences. Working with many variations of mathematics problems will increase the understanding skill in mathematics. Teachers disagreed that mathematics is one of the difficult subjects. The finding should be viewed as an advantage to the students. The teachers' positive approach about the difficulty of mathematics will transferred the students to be interested in learning mathematics.

V CONCLUSION

The fuzzy logic and conjoint analysis is used to measure the teachers' beliefs and students' expectations on learning mathematics. Respondents have stated their opinions about each attribute and these opinions are used for evaluation. Using preference levels the similarity degree between students' expectations and linguistic variables on learning mathematics. In similar way, the similarity degree between teachers' beliefs and linguistic variables on learning mathematics are found. It has been seen that the students were very strongly agreed the attributes A_2, A_3 with ranking 1 and 5 respectively. They strongly agreed A_5 with rank 2. They agreed all the other attributes. From that they need all those things from the lecture. Teachers were very strongly agreed the attributes A_{10}, A_9, A_8 with ranking 1,2,3 respectively. They were strongly agreed A_{11} with rank 5 and agree A_4 with rank 4. They were disagreed the attribute 'mathematics is a difficult subject'. It is the positive approach of the teachers. It transforms the students to be interest in learning mathematics.

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