New approach to Peak-to-Average Power Ratio Techniques for OFDM Signals

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***Abstract* - One of the challenging issues for Orthogonal Frequency Division Multiplexing (OFDM) system is its high Peak-to-Average Power Ratio (PAPR). It is a majordrawback of orthogonal frequency division multiplexing(OFDM)systems. Among the various PAPR reduction techniques, companding transform appears attractive for its simplicity andeffectiveness. This paper gives a new approach to PAPR reduction in OFDM using new companding algorithm.Compared with the others, the proposed algorithm offers animproved bit error rate and minimized out-of-band interferencewhile reducing PAPR effectively. Theoretical analysis and numericalsimulation are presented.**

***Keywords* - Companding, OFDM, PAPR.**

I. INTRODUCTION

Orthogonal frequency division multiplexing(OFDM) has been attracting substantial attention due toits excellent performance under severe channel condition [1].The rapidly growing application of OFDM includes WiMAX,DVB/DAB and 4G wireless systems.However, OFDM is not without drawbacks. One criticalproblem is its high peak-to-average power ratio (PAPR) [1].High PAPR increases the complexity of analog-to-digital(A/D) and digital-to-analog (D/A) converters, and lowers theefficiency of power amplifiers. Over the past decade variousPAPR reduction techniques have been proposed, such as blockcoding, selective mapping (SLM) and tone reservation, justto name a few [2]. Among all these techniques the simplestsolution is to clip the transmitted signal when its amplitudeexceeds a desired threshold. Clipping is a highly nonlinearprocess, however. It produces significant out-of-band interference(OBI).A good remedy for the OBI is the so-called companding.The technique 'soft' compresses, rather than 'hard' clips, thesignal peak and causes far less OBI. The method was firstproposed in [3], which employed the classical μ-law transformand showed to be rather effective. Since then many differentcompanding transforms with better performances have beenpublished [4]-[7].This paper proposes and evaluates a new compandingalgorithm which refers with the previous nonlinear companding technique, called exponential commanding, to reduce the PAPR of OFDM signals. The algorithm uses the special airy function and isable to offer an improved bit error rate (BER) and minimizedOBI while reducing PAPR effectively.

One of the major drawbacks of multicarrier transmission is the high peak-to-average power ratio (PAPR) of the transmit signal. If the peak transmit power is limited by either regulatory or application constraints, the effect is to reduce the average power allowed under multicarrier transmission relative to that under constant power modulation techniques. This in turn reduces the range of multicarrier transmission. Moreover, to prevent spectral growth of the multicarrier signal in the form of inter modulation among subcarriers and out-of-band radiation, the transmit power amplifier must be operated in its linear region (i.e., with a large input backoff), where the power conversion is inefficient. This may have a deleterious effect on battery lifetime in mobile applications. In many low-cost applications, the drawback of high PAPR may outweigh all the potential benefits of multicarrier transmission systems. A number of approaches have been proposed to deal with the PAPR problem.

The paper is organized as follows. In sectionII the PAPR problem in OFDM is briefly discussed. SectionIII presents the new algorithm and its theoretical analysis,followed by the performance simulation which gives the simulated results in graphical representation in Section IV. The next section ends finally with the conclusion.

II. PAPR IN OFDM

Let (0),(1), ⋯,$X(N-1)$ represent the data sequenceto be transmitted in an OFDM symbol with Nsubcarriers.

The baseband representation of the OFDM symbol is givenby:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{\frac{j2\pi nt}{N}} \qquad 0 \le t \le T, \quad (1)$$

where$T$is the duration of the OFDM symbol. Accordingto the central limit theorem, when $N$is large, both the realand imaginary parts of $x(t)$ become Gaussian distributed, eachwith zero mean and a variance of E[|$x(t)$|2]/2, and the amplitudeof the OFDM symbol follows a Rayleigh distribution.Consequently it is possible that the maximum amplitude ofOFDM signal may well exceed its average amplitude. Practicalhardware (e.g. A/D and D/A converters, power amplifiers) hasfinite dynamic range; therefore the peak amplitude of OFDMsignal must be limited.PAPR is mathematically defined as:

$$\text{PAPR} = 10\log_{10} \frac{\max[|x(t)|^2]}{\frac{1}{T}\int_0^T |x(t)|^2\, dt} \quad (\text{dB}) \quad (2)$$

It is easy to see from (2) that PAPR reduction may beachieved by decreasing the numerator max[|$x(t)$|2], increasingthe denominator (1/T)· $\int_0^T |x(t)|^2\, dt$, or both.The effectiveness of a PAPR reduction technique is measuredby the complementary cumulative distribution function(CCDF), which is the probability that PAPR exceeds somethreshold, i.e.:

CCDF = Probability(PAPR >p_0), (3)

wherep_0 is the threshold.

III. NEW COMPANDING ALGORITHM

OBI is the spectral leakage into alien channels. Quantificationof the OBI caused by companding requires the knowledgeof the power spectral density (PSD) of the companded signal.Unfortunately analytical expression of the PSD is in generalmathematically intractable, because of the nonlinear compandingtransform involved. Here we take an alternative approachto estimate the OBI. Let (x) be a nonlinear compandingfunction, and (t) = sin(ωt) be the input to the compander.

The companded signal (t) is:(t) = [(t)] = f[sin(ωt)] . (4)

Since (t) is a periodic function with the same period as (t),(t) can then be expanded into the following Fourier series:

$$y(t) = \sum_{k=-\infty}^{+\infty} c(k) e^{jk\omega t} \quad (5)$$

where the coefficients $c(k)$ is calculated as:

$$c(k) = c(-k) = \frac{1}{T}\int_0^T y(t) e^{-jk\omega t} dt \qquad T = \frac{2\pi}{\omega} \quad (6)$$

Notice that the input x in this case is a pure sinusoidal signal,any $(k) \neq 0$ for |k|>1 is the OBI produced by the nonlinearcompanding process. Therefore, to minimize the OBI, (k)must approach to zero fast enough as $k$increases. It hasbeen shown that (k) $\cdot k^{-(m+1)}$ tends to zero if $y(t)$ andits derivative up to the m-th order are continuous [8], or inother words, $c(k)$ converges at the rate of $k^{-(m+1)}$. Given anarbitrary number n, the n-th order derivative of $y(t)$, $d^n y/dt^n$,is a function of $d^i f(x)/dx^i$, (i= 1, 2, ⋯ , n), as well assin(ωt) and cos(ωt), i.e.:

$$\frac{d^n y}{dt^n} = g\left(\frac{d^n f(x)}{dx^n}, \frac{d^{n-1} f(x)}{dx^{n-1}}, \cdots, \frac{df(x)}{dx}, \sin(\omega t), \cos(\omega t)\right) \quad (7)$$

sin(ωt) and andcos(ωt) are continuous functions, $d^n y/dt^n$is continuous if and only if $d^i f(x)/dx^i$(i= 1, 2, ⋯ , n) arecontinuous. Based on this observation we can conclude:

Companding introduces minimum amount of OBI if thecompanding function (x) is infinitely differentiable.The functions that meet the above condition are the smoothfunctions.We now propose a new companding algorithm using asmooth function, namely the airy special function. The compandingfunction is as follows:

$(x) = \beta \cdot \text{sign}(x) \cdot [\text{airy}(0) - \text{airy}(\alpha \cdot |x|)]$, (8)

where airy(·) is the airy function of the first kind. αisthe parameter that controls the degree of companding (andultimately PAPR). $\beta$is the factor adjusting the average outputpower of the compander to the same level as the average inputpower:

$$\beta = \sqrt{\frac{E\left[|x|^2\right]}{E\left[|\text{airy}(0) - \text{airy}(\alpha \cdot |x|)|^2\right]}}, \quad (9)$$

where[·] denotes the expectation.

The decompanding function is the inverse of (x):

$$f^{-1}(x) = \frac{1}{\alpha} \cdot \text{sign}(x) \cdot \text{airy}^{-1}\left[\text{airy}(0) - \frac{|x|}{\beta}\right] \quad (10)$$

where the superscript -1 represents the inverse operation.Notice that the input to the decompander is a quantized signal with finite set of values. We can therefore numericallypre-compute $f^{-1}(x)$ and use table look-up to perform thedecompanding in practice.Next we examine the BER performance of the algorithm.Let (t) denote the output signal of the compander, (t) thewhite Gaussian noise. The received signal can be expressedas:

$$z(t) = y(t) + w(t). \quad (11)$$

The decompanded signal $\tilde{}(t)$ simply is:

$$\tilde{x}(t) = f^{-1}[z(t)] = f^{-1}[y(t) + w(t)] \quad (12)$$

Notice that the signal-to-noise ratio (SNR) in a typical additivewhite Gaussian noise (AWGN) channel is much greater than1. Using the first order Taylor series expansion, (12) can be approximated as follows:

$$\tilde{x}(t) \approx x(t) + \frac{df^{-1}(u)}{du}|_{u=y(t)} \cdot w(t). \quad (13)$$

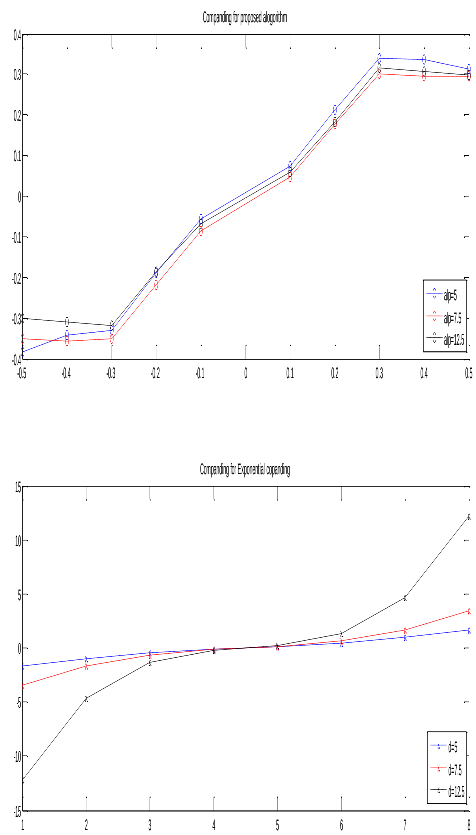

Fig. 1 Companding and decompanding profile.

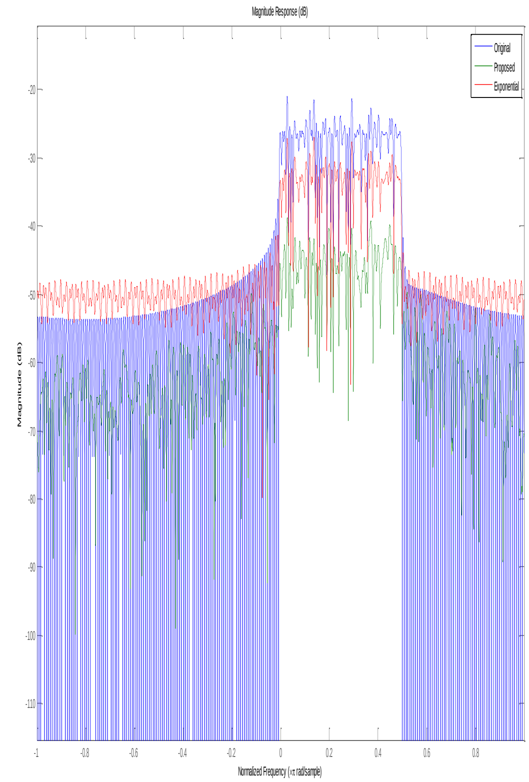

Fig. 2 Power spectral density of original and companded signals (companderinput power = 3dBm, α= 30).

Equation (13) shows that if (t) falls into the range of the decompandingfunction $f^{-1}(u)$ where $df^{-1}(u)/du|_{u=y(t)}$ <1,the noise $w(t)$ is suppressed, and if $y(t)$ is out of the range,$df-1(u)/du|_{u=y(t)}$>1 and the noise is enhanced. Therefore,if the parameter αin (8) is properly chosen such that more(t) is within the noise-suppression range of $f-1(u)$, it ispossible to achieve better overall BER performance. It is worthto mention though that BER and PAPR affect each otheradversely and therefore there is a tradeoff to make.

IV. PERFORMANCE SIMULATION

The OFDM system used in the simulation consists of 64QPSK-modulated data points. The size of the FFT/IFFT is256, meaning a 4× oversampling. Given the compander input power of 3dBm, the parameter αin the companding function is chosen to be 30.

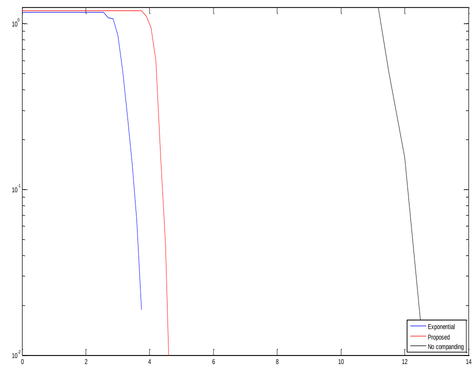

Fig. 3 Complementary cumulative distribution function of original andcompanded signals (compander input power = 3dBm, α= 30).

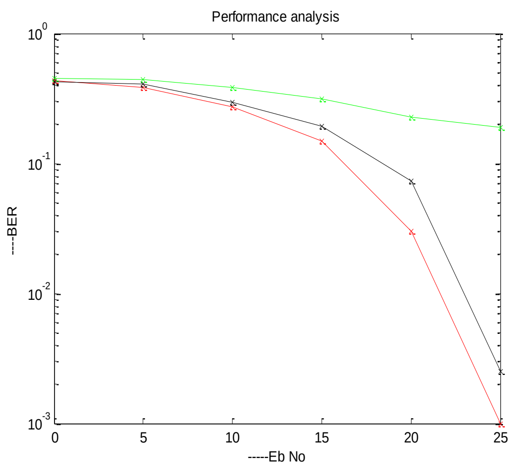

Fig. 4 Bit error rate vs. SNR for original and companded signals in AWGNchannel (compander input power = 3dBm, α= 30).

Consequently about 19.6 percent of $(t)$is within the noise-suppression range of the decompandingfunction. Two other popular companding algorithms, namelythe μ-law companding [3] and the exponential companding[5], are also included in the simulation for the purpose ofperformance comparison. The simulated PSD of the companded signals is illustratedin Fig. 2. The proposed algorithm produces OBI almost 3dBlower than the exponential algorithm, 10dB lower than theμ-law. The result is in line with our expectation. The μ-lawfunction has a singularity in its second order derivative at x =0 and therefore is expected to have the strongest OBI.Fig. 3 depicts the CCDF of the three companding schemes.The new algorithm is roughly 1.5dB inferior to the exponential,but surpasses the μ-law by 2dB.

The BER vs. SNR is plotted in Fig. 4. Our algorithm outperformsthe other two. To reach a BER of 10−3, for example,the required SNR are 8.9dB, 10.4dB and 11.7dB respectivelyfor the proposed, the exponential and the μ-law compandingschemes, implying a 1.5dB and 2.8dB improvement with thenew algorithm. The amount of improvement increases as SNRbecomes higher.One more observation from the simulation is: unlike theexponential companding whose performance is found almostunchanged under different degrees of companding, the newalgorithm is flexible in adjusting its specifications simply bychanging the value of αin the companding function.Simulation results have shown that our companding algorithm could offer better system performance in terms of PAPR reduction, power spectrum, BER, and phase error than the μ-law companding scheme. Detailed description is shown below in terms of graphical representation.

VI. CONCLUSION

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. In this we have implemented a new approach by using a new companding algorithm.Both theoretical analysis and computer simulation showthat the algorithm offers improved performance in terms ofBER and OBI while reducing PAPR effectively.

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